Integration By Parts:

When you rush through a problem with both U and V as a variable

Wish me luck on this integration by parts test guys

When you integrate by parts without writing what u and v are

Table of Contents

This is a long worksheet to cater for students that want extra practice. If you want a shortcut, but still be sure to cover one of each type then follow the pink highlighted questions.

1 Bronze

1.1 Parts Once

1.1.1 Power and Trigonometry

1) $\int x \sin 2x \, dx$
2) $\int x \cos \frac{x}{2} dx$
3) $\int_0^{\frac{\pi}{2}} x \sin 3x dx$

1.1.2 Power and Exponential

4) $\int x e^{4x} dx$
5) $\int_0^1 x e^{2x} dx$

1.1.3 Power and Ln

6) $\int x \ln 2x dx$
7) $\int_1^e x^2 \ln x \, dx$
8) $\int_1^2 \frac{1}{x^3} \ln x \, dx$

1.1.4 Power And Power

9) $\int x(x+2)^5 dx$

2 Silver

10) $\int_0^8 (4xe^{-\frac{1}{3}x} + 3)dx$. Find the exact value

11) $\int_0^2 (2e^{2x} - xe^{2x})dx$. Show that this is $\frac{1}{4}e^4 - \frac{5}{4}$

2.1.1 Power and Ln

12) $\int_1^4 x^{\frac{1}{2}} \ln 2x \, dx$. Give your answer in the form $aln2 + b$

13) $\int_1^3 (x-1) \ln x \, dx$. Show that the exact value is $\frac{3}{2} ln3$

14) $\int_1^8 \frac{1}{\sqrt[3]{x}} \ln x \, dx$. Give your answer in the form $aln2 + b$

15) Show that $\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$, where a and b are rational constants to be found

2.1.2 Power and Power

16) $\int_{-1}^1 5x\sqrt{2-x}dx$. Show that this equals $\frac{1}{3}(6\sqrt{3} - 14)$

17) $\int_1^5 (x-1)\sqrt{5-x}dx$

18) $\int_0^2 2x\sqrt{x+2}dx$. Show that this is equal to $\frac{32}{15}(2+\sqrt{2})$

2.2 Parts More Than Once (Twice)

19) $\int x^2 e^{3x} dx$

20) $\int x^2 cos2x dx$

21) $\int_0^1 x^2 e^x \, dx$

3 Gold

3.1 Parts Once

3.1.1 Power and Inverse Trig

22) $\int x^2 \tan^{-1} x \, dx$

23) $\int 2x^3 \tan^{-1} x \, dx$

3.1.2 Power and Ln

24) $\int_0^1 x \, ln(x+1) dx$

25) $\int_0^{\frac{\pi}{2}} x(4cos^2 x - 3sin^2 x) \, dx$

3.2 Parts More Than Once (Three time)

26) $\int x^3 e^x dx$

3.3 "Hidden" Parts

3.3.1 Natural Log

27) $\int \ln x \, dx$

28) $\int ln\frac{x}{2} dx$

29) $\int \ln 2x \, dx$

30) $\int_1^2 \ln 4x \, dx$

3.3.2 Inverse Trig

31) $\int \arctan x \, dx$

32) $\int \arctan 3x \, dx$

33) $\int_0^{0.5} arcsinx \, dx$

34) $\int_0^1 arccos\frac{x}{2} dx$. Show that this is $\frac{\pi}{3} + 2 - \sqrt{3}$

4 Diamond

4.1 Parts & Substitution Combined

35) $\int_0^{0.5} xln(x^2+1)dx$

36) $\int_1^4 e^{\sqrt{x}}dx$

37) $\int_0^5 e^{\sqrt{3x+1}}\,dx\ \left(u=\sqrt{3x+1}\right)$

38) $\int_0^{\sqrt{2}} x^3\ln(x^2+2)\,dx\ \ (u=x^2+2)$

39) $\int xe^{\sqrt{x}}\,dx$

4.2 Cyclic/Infinite Parts

40) $\int e^x sinx\,dx$

41) $\int e^x cos3xdx$

42) $\int e^{3x} sin2xdx$

43) $\int \frac{1}{x}\ln x\,dx$